Ninth Edition

BUSINESS MATHEMATICS in CANADA

F. ERNEST JEROME
TRACY WORSWICK

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NINTH EDITION

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F. Ernest Jerome

Tracy Worswick
Conestoga College

Mc Graw Hill Education

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About the Authors

F. Ernest Jerome received a B.Sc. degree in Honours Physics from McMaster University, where he was that university's first undergraduate to be a prize-winner in the annual Canadian Association of Physicists' Examination national competition. After earning a graduate degree in Oceanography at the University of British Columbia, he was appointed head of the Physics Department at Vancouver Island University (VIU) in Nanaimo, BC. Professor Jerome later obtained an MBA in finance from UBC, where he was awarded the Schulich Fellowship for Entrepreneurship. He subsequently taught courses in business mathematics, corporate finance, personal financial planning, mutual funds, and securities analysis in VIU's Faculty of Business. He holds a Chartered Financial Planner designation, and received the 1987 Outstanding Achievement Award from the Canadian Institute of Financial Planning.

Tracy Worswick holds degrees in Mathematics from the University of Waterloo and Mathematics Education from the University of Western Ontario. Tracy's passion for teaching and desire to deliver quality mathematics education stems from over 30 years of experience teaching Mathematics, Statistics, Computer Science, Business, and Marketing Research courses in high schools and colleges in Ontario and Alberta. For the past 18 years, she has taught Mathematics and Statistics in Conestoga College's School of Business in Kitchener, Ontario.

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Preface

Most business administration programs in Canadian colleges include an introductory course in business mathematics or mathematics of finance. *Business Mathematics in Canada* is intended for use in such courses. The text's primary objective is to support the learning of mathematics (other than statistics) needed to succeed in fields such as accounting, finance, management, marketing, personal financial planning, and business information systems.

This book may be adapted to either a one- or a two-semester course in business mathematics. It is suitable for courses that emphasize either an algebraic approach or a pre-programmed financial calculator approach to compound interest problems. (Optional spreadsheet templates provide a third alternative in many areas for students who have a basic familiarity with Microsoft Excel software.) Both algebraic solutions and financial calculator solutions are presented in most example problems for compound interest topics.

NEW IN THE NINTH EDITION

The ninth edition contains numerous changes reflecting input from faculty across the country, through reviews as well as invaluable suggestions from users of the eighth edition.

New and Updated Examples The worked examples now include **20** new problems with full solutions; another **50** examples have been updated to reflect current rates and prices.

New Calculator-Free Problems and New and Updated Exercises Calculator-free problems are new in Chapters 1 through 4 and provide exercises where students can focus primarily on the problem-solving process. The Exercises contain **100** new problems, and another **100** problems have been updated to reflect current rates and prices.

Updated Tables and Charts The exposition, tables, and example problems incorporate the most recent data at the time of writing.

Solving Two Equations and Two Unknowns has been moved to Chapter 5 with Applications of Linear Equations.

Return on Investments has been moved to Chapter 2 with the Basic Percentage Problem and Percent Change.

New Tips and Traps The ninth edition features **7** new Tips and Traps—on "Common Rounding Errors" and "Determining the Weight Factor" in Chapter 1; "The Immediate-Left Trick" and "Excel Applications in Business Mathematics" in Chapter 2; "How Low Does It Go?" and "Always Use Given Information" in Chapter 3; and "'Cost' Is Not the Selling Price" in Chapter 4.

Revised Cases and Points of Interest These features have been revised and refreshed to incorporate recent developments and time-sensitive data. The ninth edition features **7** new Points of Interest: "Following the Ups and Downs of Gas Prices in Canada" in Chapter 2, which looks at percent increases and decreases at the gas pumps; "Markup versus Margin" in Chapter 4, which examines the main differences between the two concepts; "Which Years Are Leap Years?" in Chapter 6, which explains various techniques for determining leap years; "Got a Million Dollar Talent?" in Chapter 10, which investigates the real time value of a $1M prize; "What Is Your Net Worth?" in Chapter 11, which examines this measure of personal wealth and provides a template for students to calculate their own net worth; "Rent to Own Real Estate" in Chapter 12, which explains the basics of rent to own real estate deals; and "No-Money-Down Mortgages Are No More" in Chapter 14, which summarizes the changes in Canadian mortgage rules over the past 10 years.

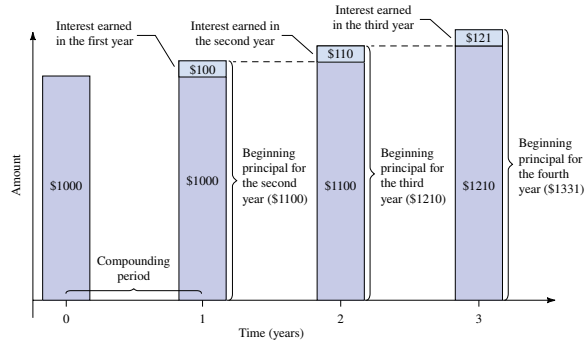
PEDAGOGICAL FEATURES

Canadian Applications Throughout the exposition, Example problems, Exercise problems, and Points of Interest, the book presents a wide range of applications of mathematics in Canadian business and finance. Every effort has been made to reflect current practices. Real financial instruments and real economic data are frequently used.

Wide Selection of Problems Each section of a chapter is followed by a set of problems for applying and reinforcing the new material. The text contains over 2000 problems and concept questions. Questions are organized by "calculator-free," "basic," "intermediate," and "advanced." Considerable effort has been made to create problems that are instructive, practical, realistic, and interesting.

Graphs and Diagrams This text makes extensive use of graphs, diagrams, and interactive charts.

FIGURE 8.1 Converting Interest to Principal at the End of Each Compounding Period



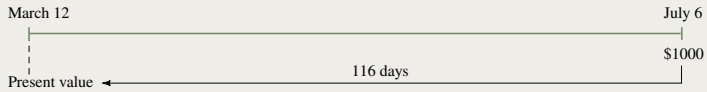
Solved Example Problems These examples provide detailed illustrations and applications of concepts in a step-by-step format.

EXAMPLE 6.4B **CALCULATING AN EQUIVALENT PAYMENT AT AN EARLIER DATE**

What is the value of a payment to be received on March 12 if it is economically equivalent to a $1000 payment originally due on the subsequent July 6, if money is worth 6.8% per year?

SOLUTION

Since we want an equivalent payment at an earlier date, we should calculate the *present* value of $1000 on March 12.



The number of days in the interval is

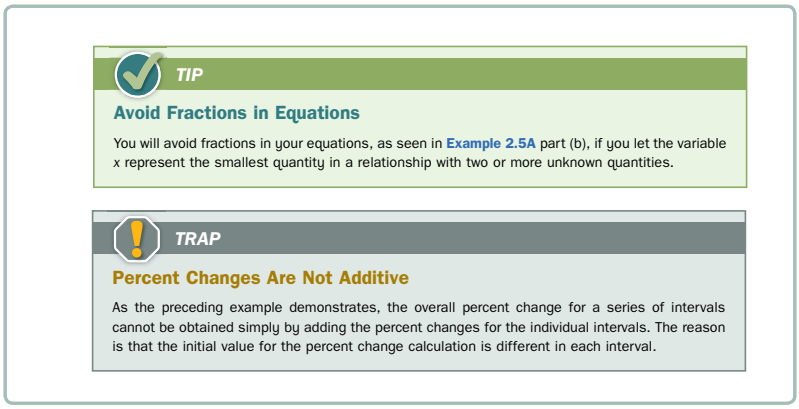
$$20 \text{ (for March)} + 30 + 31 + 30 + 5 \text{ (for July)} = 116$$

Substituting $S = \$1000$, $r = 6.8\%$, and $t = \frac{116}{365}$ into Formula (6-3), we obtain

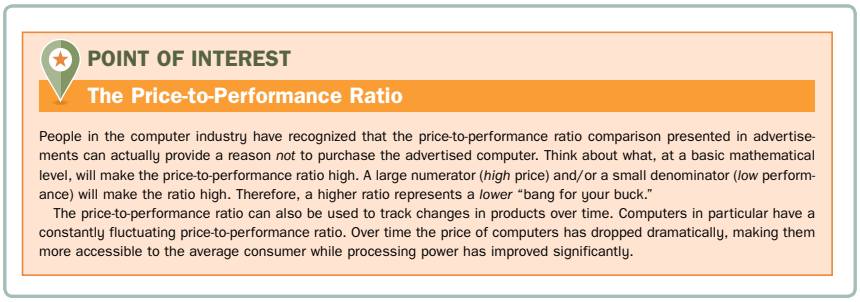
$$P = \frac{S}{1 + rt} = \frac{\$1000}{1 + 0.068\left(\frac{116}{365}\right)} = \$978.85$$

$978.85 on March 12 is equivalent to $1000 on the subsequent July 6.

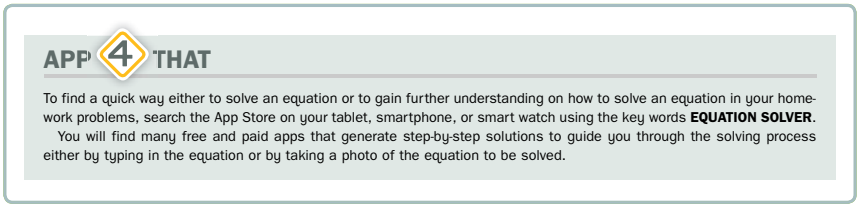
Tips and Traps Boxed elements inserted at appropriate points draw the student's attention to simplifications, pitfalls, shortcuts, calculator procedures, and common errors.



Point of Interest Boxes Most chapters contain two or three intriguing illustrations of the application or misapplication of mathematics to business and personal finance. See the Points of Interest list that follows the table of contents.



App 4 That Each boxed feature provides key words that are searchable on a smart device to find free and paid apps relevant to the topic under discussion.



Calculator Callout Boxes Many compound interest calculations can be performed using a calculator's financial functions. In the solutions for Example problems, we employ callout boxes: (1) to provide a clear visual indication of the algebraic computations that may be executed using the calculator's pre-programmed financial functions; and (2) to present the keystroke operations for employing the financial functions.

EXAMPLE 10.2A **THE FUTURE VALUE OF PERIODIC INVESTMENTS**

Heinz has been contributing $300 at the end of each month for the past 15 months to a savings plan that earns 6% compounded monthly. What amount will he have one year from now if he continues with the plan?

SOLUTION

The total amount will be the future value of $n = 15 + 12 = 27$ contributions of $PMT = \$300$ each. Payments and compounding both occur at one-month intervals. Therefore, the payments form an ordinary simple annuity having $i = \frac{6\%}{12} = 0.5\%$ per month.

$$FV = PMT\left[\frac{(1+i)^n - 1}{i}\right]$$
$$= \$300\left[\frac{(1.005)^{27} - 1}{0.005}\right]$$
$$= \$300\left(\frac{1.14415185 - 1}{0.005}\right)$$
$$= \$8649.11$$

6 I/Y
P/Y 12 ENTER
(making $C/Y = P/Y = 12$)
27 N
0 PV
300 +/– PMT
CPT FV
Ans: 8649.11

One year from now, Heinz will have $8649.11 in the plan.

Highlighted Concepts Throughout the book, statements of key concepts are highlighted, signalling to students the importance of the concept or principle.

These ideas are so important and of such wide application in finance that they are formally embodied in the Valuation Principle.

Valuation Principle
The fair market value of an investment is the sum of the present values of the cash flows expected from the investment. The discount rate used in the present value calculations should be the prevailing market-determined rate of return on this type of investment.

If the expected cash flows are received as forecast, the investor's actual rate of return on the amount invested will be precisely the discount rate used in the fair market value calculation.

Spreadsheet Templates Example problems and exercises indicated with a spreadsheet icon will direct students to an **optional** Microsoft Excel spreadsheet available on Connect. Each spreadsheet either demonstrates a solution of the example problem solved previously by algebraic or financial calculator methods or provides an alternative platform for solving the exercise problems. The spreadsheet is based on a pre-labelled and pre-formatted template.

TIP

Excel Applications in Business Mathematics

If you have not covered Section 2.8 in your course, go back and read the TIP box titled "Excel Applications in Business Mathematics" located in that section. It explains how Connect provides spreadsheet applications as an optional feature.

EXAMPLE 4.1E **CALCULATING THE NET PRICE AFTER MULTIPLE DISCOUNTS**

WGW Manufacturing and Ace Clothing both produce basic work shirts that are very similar in quality and popularity. Both manufacturers quote a list price of $46.00 for the shirt. WGW offers a regular trade discount of 25% plus an additional volume discount of 10% on orders of at least 1000 shirts. Ace offers a standard discount of 30% and a further 5% discount on orders exceeding 500 shirts. Which source will give the lower net price on an order for 1000 shirts? How much lower per shirt?

SOLUTION

Given: For WGW, $L = \$46.00$, $d_1 = 25\%$, $d_2 = 10\%$
For Ace, $L = \$46.00$, $d_1 = 30\%$, $d_2 = 5\%$

The net price per shirt from WGW is

Cases Some chapters include a case study in the end-of-chapter material. These cases usually call on concepts and skills from previous chapters as well as the current chapter.

CASE Calculations for an Investment Portfolio

One year ago, Jasmin and Derek opened investment accounts with a discount broker. In their C$ account, they purchased 300 Bank of Montreal (BMO) shares at C$54.20 per share and six Government of Canada bonds (GoCs) at C$1063 per bond. In their US$ account, they purchased 100 shares of International Business Machines (IBM) at US$125.50 per share and 200 shares of General Electric (GE) at US$18.57 per share. The exchange rate on the date of the purchases was C$1.00 = US$0.935.

The income received from the securities during the year and their current prices are listed in the following table.

	Number owned	Income received	Current price
BMO	300 shares	C$3.20 per share	C$58.15
GoCs	6 bonds	C$70.00 per bond	C$1021.50
IBM	100 shares	US$2.50 per share	US$132.25
GE	200 shares	US$0.50 per share	US$20.38

The current exchange rate is C$1.00 = US$0.952

QUESTIONS

1. What total amount in C$ was initially invested in the portfolio?
2. What is the C$ equivalent of the total income received during the year? (For converting the US$ income to C$, use the average of the beginning and ending exchange rates for the year.)
3. What is the current total value in C$ of the securities (not including income received)?
4. Including both income and change in value of the securities, what was the percentage increase in the value of Jasmin's and Derek's portfolios during the year?

Interactive Charts Through the online Connect platform, students can access interactive charts. Various Exercise problems (flagged by a "Connect" icon) invite the student to undertake an activity using an interactive chart. The student can change key variables and observe a graphic representation of the effect on the dependent variable (such as break-even point, present value, future value, market value, interest paid, amortization period, etc.).

End-of-Chapter Problems Each chapter ends with a comprehensive set of Review Problems covering the full range of topics and applications in the chapter.

Concept Questions Concept Questions are presented at the end of many sections. These questions exercise students' intuition and test their understanding of concepts and principles.

MARKET LEADING TECHNOLOGY

Learn without Limits

McGraw-Hill Connect® is an award-winning digital teaching and learning platform that gives students the means to better connect with their coursework, with their instructors, and with the important concepts that they will need to know for success now and in the future. With Connect, instructors can take advantage of McGraw-Hill's trusted content to seamlessly deliver assignments, quizzes, and tests online. McGraw-Hill Connect is a learning platform that continually adapts to each student, delivering precisely what they need, when they need it, so class time is more engaging and effective. Connect makes teaching and learning personal, easy, and proven.

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The Connect Instructor Library is a repository for additional resources to improve student engagement in and out of the class. It provides all the critical resources instructors need to build their course.

- Access Instructor resources.
- View assignments and resources created for past sections.
- Post your own resources for students to use.

Instructor Resources

- **Instructor's Solutions Manual**. Prepared by the author, with a technical review by Sarah Chan.
- **Computerized Test Bank**. Prepared by Sarah Chan.
- **Microsoft® PowerPoint® Lecture Slides**. Prepared by Rob Sorenson of Camosun College.

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For more information, please visit us online: *http://www.mheducation.ca/he/solutions*

ACKNOWLEDGMENTS

The ninth edition represents the compound future value of contributions from many dedicated educators over the previous editions. A new debt of gratitude is owed to professors who participated in a review or worked on supplements for this edition. Their thoughtful comments and suggestions have made this a better text.

Our reviewers:

Lisa MacKay	*SAIT Polytechnic*
Erica Colter	*NSCC*
Sylvia Leskiw	*MacEwan University*
Pamela Quon	*Athabasca University*
Deborah Sauer	*Capilano University*
John Molendyk	*Capilano University*
James Hebert	*Red River College*
Asifa Aamir	*UOIT*
Rob Sorensen	*Camosun College*

I am especially indebted to those who have served as a Technical Reviewer or who have been instrumental in the preparation of supplemental resources for the book. Thanks in this regard go to Sarah Chan and Rob Sorenson.

I wish to thank the staff at McGraw-Hill Ryerson for their ongoing support, professionalism, and guidance: Sara Braithwaite (Product Manager), Erin Catto (Product Developer), Joanne Limebeer (Supervising Editor), Karen Fozard (Portfolio and Program Manager), and all others involved in the development and production of this edition.

I have been fortunate during the writing of this ninth edition to be teaching many of the topics at the same time as their writing. I wish to thank all of my students, past and present, for their valuable feedback, providing insight into the content of this edition through their eyes.

Finally, I send my love and thanks to my husband Mike and family members Leslie, John, Kasey, and Jeff, who are always there to support and encourage me and for providing a lot of take-out meals when deadlines were near. Thanks also go to Chloe and Lola for tirelessly sleeping under my desk (at the risk of being rolled over by my office chair) as this ninth edition came to life.

Tracy Worswick

Chapter 1

Review and Applications of Basic Mathematics

CHAPTER OUTLINE

(Sections and chapters preceded by an asterisk* may be omitted without loss of continuity.)

LEARNING OBJECTIVES

After completing this chapter, you will be able to:

- **LO1** Perform arithmetic operations in their proper order
- **LO2** Convert fractions to their percent and decimal equivalents
- **LO3** Maintain the proper number of digits in calculations
- **LO4** Perform calculations using fractions, decimals, and percents
- **LO5** Calculate the gross earnings of employees paid a salary, an hourly wage, or commissions
- **LO6** Calculate the simple average or weighted average (as appropriate) of a set of values
- **LO7** Perform basic calculations for the Goods and Services Tax, Harmonized Sales Tax, provincial sales tax, and real property tax

MATHEMATICS PLAYS A SIGNIFICANT ROLE in business. Clients and employers now expect higher education and outstanding performance from all levels of employees that includes a significant level of mathematics competency.

Even though most routine calculations in business are done electronically, the mathematics and statistics you study in your business program are more widely expected and more highly valued in business than ever before. As a successful business graduate you must know which information is relevant, which analyses or calculations should be performed, how to interpret the results, and how to explain the outcome in terms your clients and colleagues can understand.

Naturally, a college course in business mathematics or statistics will cover a broader range of topics (often in greater depth) than you might need for a particular industry. This broader education opens more career options to you and provides a stronger set of mathematical skills for your chosen career.

TIP

How to Succeed in Business Mathematics

Since various Business Mathematics and Mathematics of Finance courses start at different points in the book, this Tip appears at the beginning of each of the first four chapters. Connect has a guide entitled "How to Succeed in Business Mathematics." Read its first two sections (A.1 and A.2) before you finish **Chapter 1**.

1.1 Order of Operations

LO1 When evaluating an expression such as

$$5 + 3^2 \times 2 - 4 \div 6$$

there is potential for confusion about the sequence of mathematical steps. Do we just perform the indicated operations in a strict left-to-right sequence called *chaining*, or is some other order intended? To eliminate any possible confusion, mathematicians have agreed on the algebraic operating system (AOS), which sets out the rules for the use of brackets and the order of mathematical operations. The rules are:

Rules for Order of Operations

1. Perform operations within brackets (in the order of Steps 2, 3, and 4 below).
2. Evaluate the powers.[1]
3. Perform multiplication and division in order from left to right.
4. Perform addition and subtraction in order from left to right.

TIP

Remembering the Order of Operations: BEDMAS

To help remember AOS, the order of operations, you can use the acronym "BEDMAS" representing the sequence: **B**rackets, **E**xponents, **D**ivision and **M**ultiplication, **A**ddition and **S**ubtraction.

[1] A power is a quantity such as 3^2 or 5^3 (which are shorthand methods for representing 3×3 and $5 \times 5 \times 5$, respectively). Section 2.2 includes a review of powers and exponents.

EXAMPLE 1.1A EXERCISES ILLUSTRATING THE ORDER OF MATHEMATICAL OPERATIONS

a. $30 - 6 \div 3 + 5$
$= 30 - 2 + 5$
$= 33$
Do division before subtraction and addition.

b. $(30 - 6) \div 3 + 5$
$= 24 \div 3 + 5$
$= 8 + 5$
$= 13$
Do operations within brackets first; then do division before addition.

c. $\frac{30 - 6}{3 + 5} = \frac{24}{8} = 3$
Brackets are implied in the numerator and the denominator.

d. $72 \div (3 \times 2) - 6$
$= 72 \div 6 - 6$
$= 12 - 6$
$= 6$
Do operations within brackets first; then do division before subtraction.

e. $72 \div (3 \times 2^2) - 6$
$= 72 \div (3 \times 4) - 6$
$= 72 \div 12 - 6$
$= 6 - 6$
$= 0$
Do operations within brackets (the power before the multiplication); then do division before subtraction.

f. $72 \div (3 \times 2)^2 - 6$
$= 72 \div 6^2 - 6$
$= 72 \div 36 - 6$
$= 2 - 6$
$= -4$
Do operations within brackets first, then the power, then divide, then subtract.

g. $4(2 - 5) - 4(5 - 2)$
$= 4(-3) - 4(3)$
$= -12 - 12$
$= -24$
Do operations within brackets first, then multiplication, then subtract.

EXERCISE 1.1

Answers to the odd-numbered problems are at the end of the book.

CALCULATOR-FREE PROBLEMS

Evaluate each of the following.

a. $10 + 10 \times 0$

b. $2 \times 2 + 4 - 8$

c. $(10 + 10) \times 0$

d. $2 \times (2 + 4) - 8$

e. $0 + 3 \times 3 - 3^2 + 10$

f. $12 - 2 \times 5 + 2^2 \times 0$

g. $0 + 3 \times 3 - (3^2 + 10)$

h. $(12 - 2) \times (5 + 2^2) \times 0$

i. $\frac{2^2 - 4}{(4 - 2)^2}$

j. $\frac{(2 - 4)^2}{5 - 2^2}$

BASIC PROBLEMS

Evaluate each of the following. In Problems 17–22, evaluate the answers accurate to the cent.

1. $20 - 4 \times 2 - 8$

2. $18 \div 3 + 6 \times 2$

3. $(20 - 4) \times 2 - 8$

4. $18 \div (3 + 6) \times 2$

5. $20 - (4 \times 2 - 8)$

6. $(18 \div 3 + 6) \times 2$

7. $54 - 36 \div 4 + 2^2$

8. $(5 + 3)^2 - 3^2 \div 9 + 3$

9. $(54 - 36) \div (4 + 2)^2$

10. $5 + (3^2 - 3)^2 \div (9 + 3)$

11. $\dfrac{8^2 - 4^2}{(4 - 2)^3}$

12. $\dfrac{(8 - 4)^2}{4 - 2^3}$

13. $3(6 + 4)^2 - 5(17 - 20)^2$

14. $(4 \times 3 - 2)^2 \div (4 - 3 \times 2^2)$

15. $[(20 + 8 \times 5) - 7 \times (-3)] \div 9$

16. $5[19 + (5^2 - 16)^2]^2$

17. $\$100\left(1 + 0.06 \times \frac{45}{365}\right)$

18. $\dfrac{\$200}{1 + 0.09 \times \frac{4}{12}}$

19. $\dfrac{\$500}{(1 + 0.05)^2}$

20. $\$1000(1 + 0.02)^3$

21. $\$100\left[\dfrac{(1 + 0.04)^2 - 1}{0.04}\right]$

22. $\$300\left[\dfrac{1 - \dfrac{1}{(1 + 0.03)^2}}{0.03}\right]$

1.2 Fractions, Decimals, and Percents

Definitions

In the fraction $\frac{3}{4}$, the upper number (3) is called the **numerator** (or dividend) and the lower number (4) is the **denominator** (or divisor). In a **proper fraction**, the numerator is smaller than the denominator. An **improper fraction** has a numerator that is larger than the denominator. A **mixed number** contains a whole number plus a fraction. **Equivalent fractions** are fractions that are equal in value (even though their respective numerators and denominators differ). An equivalent fraction can be created by multiplying or dividing both the numerator and the denominator by the same number.

EXAMPLE 1.2A EXAMPLES OF TYPES OF FRACTIONS

a. $\frac{6}{13}$ is a proper fraction.

b. $\frac{17}{13}$ is an improper fraction.

c. $2\frac{4}{13}$ is a mixed number.

d. $\frac{5}{13}, \frac{10}{26}, \frac{15}{39}$, and $\frac{20}{52}$ are equivalent fractions. Note that the second, third, and fourth fractions may be obtained by multiplying *both* the numerator and the denominator of the first fraction by 2, 3, and 4, respectively.

EXAMPLE 1.2B **CALCULATING AN EQUIVALENT FRACTION**

Find the missing numbers that make the following three fractions equivalent.

$$\frac{7}{12} = \frac{56}{?} = \frac{?}{300}$$

SOLUTION

To create a fraction equivalent to $\frac{7}{12}$, *both* the numerator and the denominator must be multiplied by the same number. To obtain 56 in the numerator of the second equivalent fraction, 7 was multiplied by 8. Hence, the denominator must also be multiplied by 8. Therefore,

$$\frac{7}{12} = \frac{7 \times 8}{12 \times 8} = \frac{56}{96}$$

To obtain the denominator (300) in the third equivalent fraction, 12 was multiplied by $\frac{300}{12} = 25$.

The numerator must also be multiplied by 25. Hence, the equivalent fraction is

$$\frac{7 \times 25}{12 \times 25} = \frac{175}{300}$$

In summary,

$$\frac{7}{12} = \frac{56}{96} = \frac{175}{300}$$

Decimal and Percent Equivalents

LO2 In the fraction $\frac{3}{4}$, the denominator indicates the total number of parts or pieces and the numerator shows how many of the parts we are considering. In other words, $\frac{3}{4}$ is 3 of 4 parts.

The *decimal equivalent* value of a fraction is obtained by dividing the numerator by the denominator. The fraction $\frac{3}{4}$ then becomes the decimal equivalent 0.75, indicating 0.75 parts of one whole piece.

To express the fraction in *percent equivalent* form, multiply the decimal equivalent by 100 (shift the decimal point two places to the right) and add the % symbol indicating parts of 100. The fraction $\frac{3}{4}$ then becomes 75%, indicating 75 parts of 100 parts.

EXAMPLE 1.2C **FINDING THE DECIMAL AND PERCENT EQUIVALENTS OF FRACTIONS AND MIXED NUMBERS**

Convert each of the following fractions and mixed numbers to its decimal equivalent and percent equivalent values.

a. $\frac{2}{5} = 0.4 = 40\%$

b. $\frac{5}{2} = 2.5 = 250\%$

c. $2\frac{3}{4} = 2.75 = 275\%$

d. $\frac{5}{8} = 0.625 = 62.5\%$

e. $1\frac{3}{16} = 1.1875 = 118.75\%$

f. $\frac{3}{1500} = 0.002 = 0.2\%$

TIP

Adding or Subtracting Fractions

To add or subtract any but the simplest of fractions, the easiest approach is to first convert each fraction to its decimal equivalent value. Then add or subtract the decimal equivalents as required.

For example, $\frac{5}{12} + \frac{23}{365} = 0.41667 + 0.06301 = 0.4797$ to four-figure accuracy.

Rounding of Decimal and Percent Equivalents

LO3 For some fractions, the decimal equivalent has an endless series of digits. Such a number is called a *nonterminating decimal*. In some cases a nonterminating decimal contains a repeating digit or a repeating group of digits. This particular type of nonterminating decimal is referred to as a *repeating decimal*. A shorthand notation for repeating decimals is to place a horizontal bar over the first occurrence of the repeating digit or group of digits. For example,

$$\frac{2}{9} = 0.222222 = 0.\overline{2} \qquad \text{and} \qquad 2\frac{4}{11} = 2.36363636 = 2.\overline{36}$$

When a nonterminating decimal or its percent equivalent is used in a calculation, the question arises: How many figures or digits should be retained? The following rules provide sufficient accuracy for the vast majority of our calculations.

Rules for Rounding Numbers

1. In intermediate results, keep one more figure than the number of figures required in the final result. (When counting figures for the purpose of rounding, do not count leading zeros used only to properly position the decimal point.)[2]
2. If the first digit dropped is 5 or greater, increase the last retained digit by 1.
3. If the first digit dropped is less than 5, leave the last retained digit unchanged.

Suppose, for example, the answer to a calculation is expected to be a few hundred dollars and you want the answer accurate to the cent. In other words, you require five-figure accuracy in your answer. To achieve this accuracy, the first rule says you should retain (at least) six figures in values used in the calculations. The rule also applies to intermediate results that you carry forward to subsequent calculations. The consequence of rounding can be stated in another way—if, for example, you use a number rounded to four figures in your calculations, you can expect only three-figure accuracy in your final answer.

EXAMPLE 1.2D FRACTIONS HAVING REPEATING DECIMAL EQUIVALENTS

Convert each of the following fractions to its decimal equivalent value expressed in the repeating decimal notation.

a. $\frac{2}{3} = 0.6666\ldots = 0.\overline{6}$

b. $\frac{14}{9} = 1.555\ldots = 1.\overline{5}$

c. $6\frac{1}{12} = 6.08333\ldots = 6.08\overline{3}$

d. $3\frac{2}{11} = 3.181818\ldots = 3.\overline{18}$

e. $5\frac{2}{27} = 5.074074\ldots = 5.\overline{074}$

f. $\frac{5}{7} = 0.714285714285\ldots = 0.\overline{714285}$

[2] The following example illustrates the reasoning behind this instruction. A length of 6 mm is neither more nor less precise than a length of 0.006 m. (Recall that there are 1000 mm in 1 m.) The leading zeros in 0.006 m do not add precision to the measurement. They are inserted to properly position the decimal point. Both measurements have one-figure accuracy. Contrast this case with measurements of 1007 mm and 1.007 m. Here each zero comes from a decision about *what* the digit should be (rather than *where* the decimal point should be). These measurements both have four-figure accuracy. This rule applies to the total number of figures (other than leading zeros) in a value. It does not apply to the number of *decimal* places.

EXAMPLE 1.2E CALCULATING AND ROUNDING THE DECIMAL EQUIVALENTS OF FRACTIONS

Convert each of the following fractions and mixed numbers to its decimal equivalent value rounded to four-figure accuracy.

a. $\frac{2}{3} = 0.6667$ **b.** $6\frac{1}{12} = 6.083$ **c.** $\frac{173}{11} = 15.73$

d. $\frac{2}{1071} = 0.001867$ **e.** $\frac{17{,}816}{3} = 5939$

TRAP

Common Rounding Errors

The following examples illustrate two of the most common rounding errors. Each example requires rounding to two decimal places or three significant digits:

Example 1: 2.4449 = 2.445 = 2.45 This is NOT correct.

For rounding to two decimal places, you only need to consider the third decimal place to apply Rule 3.

Here, continuous rounding has been applied starting with the first digit that is 5 or larger. The correct answer is 2.44.

Example 2: 2.992 = 2.99 = 3.00 This is NOT correct.

The value is originally rounded correctly using Rule 3 but then rounded again applying Rule 2.

Round one time only to achieve the required number of significant digits. The correct answer is 2.99.

EXAMPLE 1.2F DEMONSTRATING THE CONSEQUENCES OF TOO MUCH ROUNDING

Accurate to the cent, evaluate

$$\$140\left(1 + 0.11 \times \tfrac{113}{365}\right) + \$74\left(1 + 0.09 \times \tfrac{276}{365}\right)$$

SOLUTION

If you want five-figure accuracy in your answer, you cannot round to fewer than six figures *at any stage* of the calculations. The following table illustrates how too much rounding can result in an inaccurate answer.

If we first evaluate the contents of the brackets before rounding, we obtain: \$140(1.0340548) + \$74(1.0680548)

6-Figure Accuracy	5-Figure Accuracy	3-Figure Accuracy
\$140(1.03405) + \$74(1.06805) = \$144.767 + \$79.0357 = \$223.8027 = \$223.80 (rounded to the cent)	\$140(1.0341) + \$74(1.0681) = \$144.774 + \$79.0394 = \$223.8134 = \$223.81 (rounded to the cent)	\$140(1.03) + \$74(1.07) = \$144.20 + \$79.18 = \$223.38
Correct answer	\$0.01 larger than correct answer	\$0.42 smaller than correct answer

One more point is worth noting. Consider the first column, where you properly maintained six-figure accuracy. That is,

$$\$140(1.03405) + \$74(1.06805) = \$144.767 + \$79.0357$$

Suppose you round the two amounts on the right side to the nearest cent *before* you add them. The sum is then

$$\$144.77 + \$79.04 = \$223.81$$

which is \$0.01 larger than the correct answer. The error arises because, just at the final addition, you failed to maintain six-figure accuracy (to ensure five-figure accuracy in the final answer).

TIP

Optimal Use of Your Calculator

Whenever possible, use your calculator's memory registers to save intermediate results. This will save time and reduce keystroke errors during data re-entry. It also virtually eliminates the introduction of rounding errors, since most calculators internally retain two or three more figures than are shown in the display. **Example 1.2G** illustrates this approach.

EXAMPLE 1.2G **OPTIMAL USE OF YOUR CALCULATOR**

We will again evaluate (accurate to the cent) the same expression as in **Example 1.2F**,

$$\$140\left(1 + 0.11 \times \tfrac{113}{365}\right) + \$74\left(1 + 0.09 \times \tfrac{276}{365}\right)$$

This time we will use our financial calculator in a way that (1) avoids manual re-entry of intermediate results, and (2) maintains maximum precision by avoiding rounding (other than rounding imposed by the inherent limitations of the calculator).

SOLUTION

We assume the Texas Instruments BA II PLUS calculator is set for a floating-decimal format and for the algebraic operating system (AOS) calculation method. (Refer to **Appendix 1A** for instructions on making these settings.) In the AOS mode, we can enter numbers, brackets, and operations in the same left-to-right sequence as they are written. The calculator performs the calculations according to the proper order of operations.

140 [×] [(] 1 [+] 0.11 [×] 113 [÷] 365 [)]
[+] 74 [×] [(] 1 [+] 0.09 [×] 276 [÷] 365 [)] [=] 223.80

The result is \$223.80.

You see that it is possible to evaluate quite complex expressions without writing down intermediate results. However, if someone is going to read and readily understand your solution, you should present enough detail and intermediate results to reveal the steps in your solution.

TIP

Using a Calculator's Power Function

Use the following sequence of keystrokes to evaluate 1.62^5 with the power function key [y^x].

1.62 [y^x] 5 [=]

If the symbol y^x sits above a calculator key (rather than on it), the power function is the secondary function of the key. The keystroke sequence is then

1.62 [2nd] [y^x] 5 [=]

The answer to seven-figure accuracy is 11.15771.

Example 1.2H uses this feature.

Evaluating Complex Fractions

A **complex fraction** is a fraction containing one or more other fractions in its numerator or denominator. In simplifying complex fractions, particular attention should be paid to the correct order of mathematical operations as discussed in **Section 1.1**.

EXAMPLE 1.2H EVALUATING COMPLEX FRACTIONS

Evaluate each of the following complex fractions accurate to the cent.

a. $\dfrac{\$425}{\left(1+\frac{0.09}{12}\right)^{24}}$

b. $\dfrac{\$1265\left(1+0.115\times\frac{87}{365}\right)}{1+0.125\times\frac{43}{365}}$

c. $\dfrac{\$1}{1+0.025\times\frac{5}{12}}+\dfrac{\$1}{1+0.04\times\frac{2}{12}}$

SOLUTION

We assume the Texas Instruments BA II PLUS calculator is set for a floating-decimal format and for the algebraic operating system (AOS) calculation method. Refer to **Appendix 1A** for instructions on making these settings.

a. 425 [÷] [(] 1 [+] 0.09 [÷] 12 [)] [y^x] 24 [=] 355.23

The result is $355.23.

b. One-step method where additional brackets must be used for the denominator:

1265 [×] [(] 1 [+] 0.115 [×] 87 [÷] 365 [)]

[÷] [(] 1 [+] 0.125 [×] 43 [÷] 365 [)] [=] 1280.81

Two-step method using the calculator's memories to store intermediate answers:

1265 × (1 + 0.115 × 87 ÷ 365) = 1299.674863 STO 1
1 + 0.125 × 43 ÷ 365 = 1.014726027 STO 2
RCL 1 ÷ RCL 2 = $1280.81

The result is $1280.81

c. 1 + 0.025 × 5 ÷ 12 = 1.0104166667 1/X (0.989690722) STO 1
1 + 0.04 × 2 ÷ 12 = 1.006666667 1/X (0.993377483) STO 2
RCL 1 + RCL 2 = $1.98

The result is $1.98.

Calculating Percent of a Number

LO4 Calculating the percent of a number is one of the most common calculations in business. To find the percent of a number, convert the percent to its decimal equivalent by dividing the percent by 100 (shifting the decimal point two places to the left) and then multiplying by the number.

EXAMPLE 1.2I

a. What is 22% of \$185?

b. What is $40\frac{1}{4}\%$ of \$140.25?

c. How much is $0.08\overline{3}\%$ of \$5000?

d. How much is 140% of \$50?

SOLUTION

a. The question asks us to calculate a part (percent) of a given whole. By converting 22% to its decimal equivalent we obtain

$$\frac{22}{100} \times \$185 = 0.22 \times \$185 = \$40.70$$

That is, 22% of \$185 is \$40.70.

b. $40\frac{1}{4}\%$ of \$140.25 becomes

$$40.25\% \times \$140.25 = 0.4025 \times \$140.25 = \$56.45$$

That is, $40\frac{1}{4}\%$ of \$140.25 is \$56.45.

c. In converting $0.08\overline{3}\%$ to its decimal equivalent, we need to be careful to maintain the correct accuracy in calculations to have our answer accurate to the cent. Since 1% of \$5000 is \$50, then 0.1% of \$5000 is only \$5. Therefore, the answer will be a little less than \$5. For the answer to be accurate to the cent, we seek three-figure accuracy.

$$0.08\overline{3}\% \times \$5000 = 0.0008333 \times \$5000 = \$4.17$$

Therefore, \$4.17 is $0.08\overline{3}\%$ of \$5000.

d. Here the percentage is greater than 100% so the answer will be larger than the original value of \$50.

$$1.40 \times \$50 = \$70$$

Therefore, 140% of \$50 is \$70.

TRAP

Decimal Equivalent of Percentages Smaller Than 1%

When a percent is less than 1%, students sometimes forget to move the decimal two places to the left in order to obtain the decimal equivalent. For example, be clear on the distinction between 0.25% and 25%. The former is just $\frac{1}{4}$ of 1%—the latter is 25 *times* 1%. Their decimal equivalents are 0.0025 and 0.25, respectively. In terms of equivalent fractions, 0.25% equals $\frac{1}{400}$, but 25% equals $\frac{1}{4}$.

EXAMPLE 1.2J A WORD PROBLEM REQUIRING THE USE OF PERCENTS

A battery manufacturer encloses a rebate coupon for 15% off in a package of two AAA batteries retailing for \$6.29. What rebate does the coupon represent?

SOLUTION

In effect, the question is asking you to find 15% of the retail price.

$$\textit{Rebate} = 0.15 \times \$6.29 = \$0.94$$

The manufacturer's 15% rebate on the batteries is equivalent to a cash rebate of \$0.94.